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Fuzzy multiobjective linear model for supplier selection in a supply chain

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Abstract

Within new strategies for purchasing and manufacturing, suppliers play a key role in achieving corporate competition. Hence, selecting the right suppliers is a vital component of these strategies. In practice, vagueness and imprecision of the goals, constraints and parameters in this problem make the decision-making complicated.

In spite of the importance of the problem the literature on this subject is relatively scarce. In this paper a fuzzy multiobjective linear model is developed to overcome the vagueness of the information. For the first time in a fuzzy supplier selection problem, an asymmetric fuzzy-decision making technique is applied to enable the decision-maker to assign different weights to various criteria. The model is explained by an illustrative example.

Keywords: Supplier selection; Fuzzy multiobjective decision making; Supply chain

1. Introduction

Supplier selection is one of the most critical activities of purchasing management in a supply chain, because of the key role of supplier's performance on cost, quality, delivery and service in achieving the objectives of a supply chain.

Supplier selection is a multiple criteria decision-making (MCDM) problem which is affected by several conflicting factors. Consequently, a purchasing manager must analyze the trade off among the several criteria. MCDM techniques support the decision-makers DMs in evaluating a set of alternatives. Depending upon the purchasing situations, criteria have varying mportance and there is a need to weight criteria (Dulmin and Mininno, 2003).

In a real situation for a supplier selection problem, many input information are not known precisely. At the time of making decisions, the value of many criteria and constraints are expressed in vague terms such as "very high in quality" or "low in price". Deterministic models cannot easily take this vagueness into account. In

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these cases the theory of fuzzy sets is one of the best tools for handling uncertainty. Fuzzy set theories are employed due to the presence of vagueness and imprecision of information in the supplier selection problem.

Bellman and Zadeh (1970) suggested a fuzzy programming model for decision-making in fuzzy environments. Zimmermann (1978) first used the Bellman and Zadeh (1970) method to solve fuzzy multiobjective linear programming problems. In his model the fuzzy goals and fuzzy constraints are treated equivalently, which is why the model is called symmetric. It is very common in business activities, such as supplier selection, that the goals importance or weights are different for DMs. Thus, the symmetrical models may not be appropriate for the same multiobjective decision-making problem, because the objectives may not be equally important.

In this paper, for the first time, a fuzzy multiobjective model has been developed for the supplier selection problem, in which different weights can be considered for various objectives.

2. Literature review

The literature in this area discusses either the criteria or the methods of supplier selection.

Dickson (1966) firstly identified and analyzed the importance of 23 criteria for supplier selection based on a survey of purchasing managers. He showed that quality is the most important criterion followed by delivery and performance history. Weber et al. (1991) reviewed 74 articles discussing supplier selection criteria, and showed that net price is the most important criterion for supplier selection. They also concluded that supplier selection is a multicriteria problem and the priority of criteria depends on each purchasing situation. Roa and Kiser (1980) and Bache et al. (1987) identified, respectively, 60 and 51 criteria for supplier selection. A comprehensive review of criteria for supplier selection is presented in Ghodsypour and O'Brien (1996). He concluded that the number and the weights of criteria depend on purchasing strategies.

Gaballa (1974) is the first author who applied mathematical programming to supplier selection in a real case. He used mixed integer programming to minimize the total discounted price of allocated items to the suppliers. Formulated a single-objective, mixed-integer programming to minimize the sum of purchasing, transportation and inventory costs by considering multiple items, multiple time periods, vendors' quality, delivery and capacity. Weber and Current (1993) used a multiobjective approach to systematically analyze the trade-offs between conflicting criteria in supplier selection problems.

Ghodsypour and O'Brien (1997) developed a decision support system (DSS) for reducing the number of suppliers according to supply based optimization strategy. They used an integrated analytical hierarchy process (AHP) with mixed-integer programming and considered suppliers' capacity constraint and the buyers' limitations on budget and quality etc. Ghodsypour and O'Brien (1998) developed an integrated AHP and linear programming model to consider both qualitative and quantitative factors in purchasing activity.

Karpak et al. (1999) used a goal programming model to minimize costs and maximize delivery reliability and quality in supplier selection when assigning the order quantities to each supplier. Degraeve and Roodhooft (2000) developed a total cost approach with mathematical programming to treat supplier selection using activity-based cost information. Ghodsypour and O'Brien (2001) developed a mixed-integer non-linear programming approach to minimize total cost of logistics, including net price, storage, ordering costs and transportation in supplier selection. However, due to the vagueness of the information related to parameters, these deterministic models are unsuitable to obtain an effective solution for supplier selection problem.

In the literature, there are few papers in order to handle imprecise information and uncertainty in supplier selection models (Narasimhan, 1983, Soukup, 1987, Nydick and Hill, 1992). In these papers, for finding the best overall rating supplier, simple linear weighting models have been adapted to deal with uncertainty from incomplete and qualitative data in unstructured purchasing situations.

Based on fuzzy logic approaches, Morlacchi (1997) developed a model that combines the use of fuzzy set theory (FST) with AHP and implements it to evaluate small suppliers in the engineering and machine sectors.

Li et al. (1997) proposed a measure for supplier performance evaluation. They used fuzzy bag method to score qualitative criteria and then all scores for qualitative and quantitative criteria are combined in an intuitive sum of weighted averages. Holt (1998) reviewed of contractor evaluation and selection modeling methodologies including FST method. In these methods, binary decisions (e.g. the contractor does, or does not, have a formal safety policy) can convert to linguistic variables (e.g. No, Minimum, Strong and

Maximum). Erol and Ferrel (2003) proposed a methodology that assists DMs to use qualitative and quantitative data in a multiobjective mathematical programming model. In their method first, qualitative information converts into quantitative format using fuzzy quality function deployment (QFD) and then combines this data with other quantitative data to parameterize a multiobjective model. They discuss the problem without capacity constraint while in this paper, developed model solves the problem with capacity constraint. In other words these papers deal with single sourcing supplier selection (one supplier can satisfy all buyer's needs) where as our model discusses multiple sourcing (Ghodsypour and O'Brien, 1998).

Supplier selection is a multiobjective decision-making problem, in which criteria should have different weights. Vagueness of the information in this problem, make the decision-making complicated. In this paper, a fuzzy multiobjective model is developed to assign different weights to the various criteria. This fuzzy model enables the purchasing managers not only to consider the imprecision of information but also take the limitations of buyer and supplier into account to calculate the order quantity assigned to each supplier.

The paper is organized as follows: in Section 3 the fuzzy multiobjective model and its crisp formulation for the supplier selection problem is presented in which the objectives are not equally important and have different weights. First, a general linear multiobjective formulation for this problem is considered and then some definitions and appropriate approach for solving this decision-making problem are discussed. Section 4 presents the numerical example and explains the results. Finally, the concluding remarks are presented in Section 5.

3. The multiobjective supplier selection model

A general multiobjective model for the supplier selection problem can be stated as follows (Weber and Current 1993, Ghodsypour and O'Brien, 2001):

$$\min Z_1, Z_2, \ldots, Z_k, \tag{1}$$

$$\max\ Z_{k+1}, Z_{k+2}, \ldots, Z_P, \tag{2}$$

s. t.:

$$x \in X_d,\ X_d = \{x/g(x) \leqslant b_r,\ r = 1, 2, \ldots, m\} \tag{3}$$

where Z_1, Z_2, …,Z_k are the negative objectives or criteria-like cost, late delivery, etc. and Z_{k+1}, Z_{k+2},…,Z_p are the positive objectives or criteria such as quality, on time delivery, after sale service and so on. X_d is the set of feasible solutions which satisfy the constraint such as buyer demand, supplier capacity, etc.

A typical linear model for supplier selection problems is (Weber and Current, 1993 and Ghodsypour and O'Brien, 2001) min Z_1; max Z_2, Z_3 with

$$Z_1 = \sum_{i=1}^{n} P_i x_i, \tag{4}$$

$$Z_2 = \sum_{i=1}^{n} F_i x_i, \tag{5}$$

$$Z_3 = \sum_{i=1}^{n} S_i x_i \tag{6}$$

s.t.

$$\sum_{i=1}^{n} x_i \geqslant D, \tag{7}$$

$$x_i \leqslant C_i, \quad i = 1, 2, \ldots, n \tag{8}$$

$$x_i > 0, \quad i = 1, 2, \ldots, n \tag{9}$$

where D is demand over period, x_i is the number of units purchased from the ith-supplier, P_i is per unit net purchase cost from supplier i, C_i is capacity of ith supplier, F_i is percentage of quality level of ith supplier, S_i is percentage of service level of ith supplier, n is number of suppliers.

Three objective functions—net price (4), quality (5) and service (6)—are formulated to minimize total monetary cost, maximize total quality and service level of purchased items, respectively. Constraint (7) ensures that demand is satisfied. Constraint set (8) means that order quantity of each supplier should be equal or less than its capacity and constraint set (9) prohibits negative orders.

In a real case, DMs do not have exact and complete information related to decision criteria and constraints. For supplier selection problems the collected data does not behave crisply and they are typically fuzzy in nature. A fuzzy multiobjective model is developed to deal with the problem. Before presenting the fuzzy model, some definitions and notation should be discussed.

3.1. Definitions

Fuzzy set. Let X be a universe of discourse, A is a fuzzy subset of X if for all $x \in X$, there is a number $\mu_A(x) \in [0, 1]$ assigned to represent the membership of x to A, and $\mu_A\ (x)$ is called the membership function of A.

α-cut. The (crisp) set of elements that belong to the fuzzy set A for which the degree of its membership function exceeds the level α: $A_\alpha = [x \in X \big| \mu_A(x) \geqslant \alpha]$.

Fuzzy decision. A fuzzy decision is defined in an analogy to non-fuzzy environments "as the selection of activities which simultaneously satisfy objective functions and constraints". In fuzzy set theory the intersection of sets normally corresponds to the logical "and". The "decision" in a fuzzy environment can therefore be viewed as the intersection of fuzzy constraints and fuzzy objective functions (Zimmermann, 1978). The fuzzy decision can be divided into two categories, symmetric and asymmetric fuzzy decision-making. In a symmetrical fuzzy decision there is no difference between the weight of objectives and constraints while in the asymmetrical multi-objective fuzzy decision, the objectives and constraints are not equally important and have different weights (Zimmermann, 1978 and 1987; Sakawa, 1993).

Constructing either the symmetrical or the asymmetrical model depends upon the selection of operators. For fuzzy decision-making, the selection of appropriate operators is very important. Zimmermann (1993) classified eight important criteria that may be helpful for selecting the appropriate operators in fuzzy decisions. In the next section, the appropriate operator related to the fuzzy supplier selection problem is discussed.

3.2. The fuzzy supplier selection model

In this section, first the general multiobjective model for supplier selection is presented and then appropriate operators for this decision-making problem are discussed.

A general linear multiobjective model can be presented as:

Find a vector x written in the transformed form $x^{\mathrm{T}} = [x_1,\ x_2, \dots, X_n]$ which minimizes objective function Z_k and maximizes objective function Z_l with

$$Z_k = \sum_{i=1}^{n} c_{ki} x_i, \quad k = 1,\ 2, \dots, p, \tag{10}$$

$$Z_l = \sum_{i=1}^{n} c_{li} x_i, \quad l = p+1,\ p+2 \dots q \tag{11}$$

and constraints:

$$x \in X_d, \quad X_d = \left\{ x/g(x) = \sum_{i=1}^{n} a_{ri} x_i \leqslant b_r, \quad r = 1, 2, \dots, m,\ x \geqslant 0 \right\}, \tag{12}$$

where c_{ki}, c_{li}, a_{ri} and b_r are crisp or fuzzy values.

Zimmermann (1978) has solved problems (10–12) by using fuzzy linear programming. He formulated the fuzzy linear program by separating every objective function Z_j into its maximum Z_j^+ and minimum Z_j^- value by solving:

$$Z_k^+ = \max Z_k, \quad x \in X_a, \qquad Z_k^- = \min Z_k, \quad x \in X_d, \tag{13}$$

$$Z_l^+ = \max Z_l, \quad x \in X_d, \qquad Z_l^- = \min Z_l, \quad x \in X_a. \tag{14}$$

Z_k^-, Z_l^+ are obtained through solving the multiobjective problem as a single objective using, each time, only one objective and $x \in X_d$ means that solutions must satisfy constraints while X_a is the set of all optimal solutions through solving as single objective.

Since for every objective function Z_j, its value changes linearly from Z_j^- to Z_j^+, it may be considered as a fuzzy number with the linear membership function $\mu_{zj}(x)$ as shown in Fig. 1.

It was shown that a linear programming problem (10–12) with fuzzy goal and fuzzy constraints may be presented as follows:

Find a vector x to satisfy:

$$\tilde{Z}_k = \sum_{i=1}^{n} c_{ki} x_i \leqslant \sim Z_k^0, \quad k = 1, 2, \ldots, p, \tag{15}$$

$$\tilde{Z}_l = \sum_{i=1}^{n} c_{li} x_i \geqslant \sim Z_l^0, \quad l = p+1, p+2, \ldots, q \tag{16}$$

s.t.:

$$\tilde{g}_i(x) = \sum_{i=1}^{n} a_{ri} x_i \leqslant \sim b_r, \quad r = 1, 2, \ldots, h \qquad \text{(for fuzzy constraints)}, \tag{17}$$

$$g_p(x) = \sum_{i=1}^{n} a_{pi} x_i \leqslant b_p, \quad p = h+1, \ldots, m \quad \text{(for deterministic constraints)}, \tag{18}$$

$$x_i \geqslant 0, \quad i = 1, 2, \ldots, n. \tag{19}$$

In this model, the sign $\sim$ indicates the fuzzy environment. The symbol $\leqslant\sim$ in the constraints set denotes the fuzzified version of $\leqslant$ and has linguistic interpretation "essentially smaller than or equal to" and the symbol $\geqslant\sim$ has linguistic interpretation "essentially grater than or equal to". Z_k^0 and Z_l^0 are the aspiration levels that the decision-maker wants to reach.

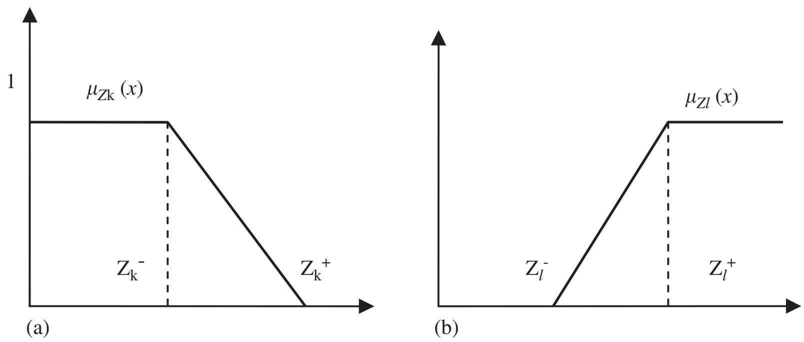

Fig. 1. Objective function as fuzzy number: (a) for minimizing objective function Z_k (negative objective) and (b) for maximizing objective function Z_l (positive objective).

Assuming that membership functions, based on preference or satisfaction are linear the linear membership for minimization goals (Z_k) and maximization goals (Z_l) are given as follows:

$$\mu_{zk}(x) = \begin{cases} 1 & \text{for } Z_k \leqslant Z_k^-, \\ (Z_k^+ - Z_k(x))/(Z_k^+ - Z_k^-) & \text{for } Z_k^- \leqslant Z_k(x) \leqslant Z_k^+, \quad k = 1, 2, \ldots, p, \\ 0 & \text{for } Z_k \geqslant Z_k^+. \end{cases} \tag{20}$$

$$\mu_{zl}(x) = \begin{cases} 1 & \text{for } Z_l \geqslant Z_l^+, \\ (Z_l(x) - Z_l^-)/(Z_l^+ - Z_l^-) & \text{for } Z_l^- \leqslant Z_l(x) \leqslant Z_l^+, \quad l = p+1, p+2, \ldots, q, \\ 0 & \text{for } Z_l \leqslant Z_l^-. \end{cases} \tag{21}$$

The linear membership function for the fuzzy constraints is given as

$$\mu_{gr}(x) = \begin{cases} 1 & \text{for } g_r(x) \leqslant b_r, \\ 1 - (g_r(x) - b_r)/d_r & \text{for } b_r \leqslant g_r(x) \leqslant b_r + d_r, \quad r = 1, 2, \ldots, h, \\ 0 & \text{for } g_r(x) \geqslant b_r + d_r. \end{cases} \tag{22}$$

d_r is the subjectively chosen constants expressing the limit of the admissible violation of the rth inequalities constraints (tolerance interval).

In the next section some important fuzzy decision-making operators will be presented.

3.3. Decision making operators

First, the max–min operator is discussed, which was used by Zimmermann (1987, 1993) for fuzzy multiobjective problems. Then, the convex (weighted additive) operator is stated that enables the DMs to assign different weights to various criteria.

In fuzzy programming modeling, using Zimmermann's approach, a fuzzy solution is given by the intersection of all the fuzzy sets representing either fuzzy objective or fuzzy constraints. The fuzzy solution for all fuzzy objectives and h fuzzy constraints may be given as

$$\mu_D(x) = \left\{ \left\{ \bigcap_{j=1}^{q} \mu_{z_j}(x) \right\} \bigcap \left\{ \bigcap_{r=1}^{h} \mu_{g_r}(x) \right\} \right\}. \tag{23}$$

The optimal solution(x^*) is given by

$$\mu_D(x^*) = \max_{x \in X_d} \mu_D(x) = \max_{x \in X_d} \min \left[\min_{j=1,..,q} \mu_{z_j}(x), \min_{r=1,..,h} \mu_{g_r}(x) \right]. \tag{24}$$

In order to find optimal solution (x^*) in the above fuzzy model, it is equivalent to solving the following crisp model (Zimmermann, 1978):

$$\text{Maximize } \lambda \tag{25}$$

s.t.:

$$\lambda \leqslant \mu_{z_j}(x), \quad j = 1, 2, \ldots, q \qquad \text{(for all objective functions)}, \tag{26}$$

$$\lambda \leqslant \mu_{g_r}(x), \quad r = 1, 2, \ldots, h \qquad \text{(for fuzzy constraints)}, \tag{27}$$

$$g_p(x) \leqslant b_p, \quad p = h+1, \ldots, m \qquad \text{(for deterministic constraints)}, \tag{28}$$

$$x_i \geqslant 0, \quad i = 1, 2, \ldots, n \quad \text{and} \quad \lambda \in [0, 1], \tag{29}$$

where $\mu_D(x)$, $\mu_{zj}(x)$ and $\mu_{gr}(x)$ represent the membership functions of solution, objective functions and constraints.

In this solution the relationship between constraints and objective functions in a fuzzy environment is fully symmetric (Zimmermann, 1978). In other words, in this definition of the fuzzy decision, there is no difference between the fuzzy goals and fuzzy constraints. Therefore, depending on the supplier selection problem, situations in which fuzzy goals and fuzzy constraints have unequal importance to DM and other patterns, as the confluence of objectives and constraints, should be considered. The weighted additive model can handle this problem, which is described as follows:

The weighted additive model is widely used in vector-objective optimization problems; the basic concept is to use a single utility function to express the overall preference of DM to draw out the relative importance of criteria (Lai and Hawang, 1994). In this case, multiplying each membership function of fuzzy goals by their corresponding weights and then adding the results together obtain a linear weighted utility function.

The convex fuzzy model proposed by Bellman and Zadeh (1970), Sakawa (1993) and the weighted additive model, Tiwari et al. (1987) is

$$\mu_D(x) = \sum_{j=1}^{q} w_j \mu_{z_j}(x) + \sum_{r=1}^{h} \beta_r \mu_{g_r}(x), \tag{30}$$

$$\sum_{j=1}^{q} w_j + \sum_{r=1}^{h} \beta_r = 1, \quad w_j,\ \beta_r \geqslant 0, \tag{31}$$

where w_j and β_i are the weighting coefficients that present the relative importance among the fuzzy goals and fuzzy constraints. The following crisp single objective programming is equivalent to the above fuzzy model:

$$\max \sum_{j=1}^{q} w_j \lambda_j + \sum_{r=1}^{h} \beta_r \gamma_r \tag{32}$$

s.t.:

$$\lambda_j \leqslant \mu_{z_j}(x), \quad j = 1, 2, \ldots, q, \tag{33}$$

$$\gamma_r \leqslant \mu_{g_r}(x), \quad r = 1, 2, \ldots, h, \tag{34}$$

$$g_p(x) \leqslant b_p, \quad p = h+1, \ldots, m, \tag{35}$$

$$\lambda_j, \gamma_r \in [0, 1], \quad j = 1, 2, \ldots, q \quad \text{and} \quad r = 1, 2, \ldots, h, \tag{36}$$

$$\sum_{j=1}^{q} w_j + \sum_{r=1}^{h} \beta_r = 1, \quad w_j, \beta_r \geqslant 0, \tag{37}$$

$$x_i \geqslant 0, \quad i = 1, 2, \ldots, n. \tag{38}$$

3.4. Model algorithm

Complete formulations of supplier selection problems to the fuzzy multiobjective are stated in the following steps:

Step 1: Construct the supplier selection model according to the criteria and constraints of the buyer and suppliers.

Step 2: Solve the multiobjective supplier selection problem as a single-objective supplier selection problem using each time only one objective. This value is the best value for this objective as other objectives are absent.

Step 3: From the results of step 2 determine the corresponding values for every objective at each solution derived.

Step 4: From step 3, for each objective function find a lower bound and an upper bound corresponding to the set of solutions for each objective. Let Z_j^- and Z_j^+ denote the lower bound and upper bound for the jth objective (Z_j) from (13) and (14).

Step 5: For the objective functions and fuzzy constraints find the membership function according to (20–22).

Step 6: From step 5 and DM's preferences, based on fuzzy convex decision-making formulate the equivalent crisp model of the fuzzy optimization problem according to (32–38).

Step 7: Find the optimal solution vector x^*, where x^* is the efficient solution of the original multiobjective supplier selection problem with the DM's preferences.

The model algorithm is illustrated through a numerical example.

4. Numerical example

For supplying a new product to a market assume that three suppliers should be managed. The purchasing criteria are net price, quality and service. The capacity constraints of suppliers are also considered.

It is assumed that the input data from suppliers' performance on these criteria are not known precisely. The de-fuzzified values of their cost, quality and service level and constraints of suppliers are presented in Table 1. The demand is a fuzzy number and is predicted to be about 1000, as shown in Table 2.

The multiobjective linear formulation of numerical example is presented as min Z_1 and max Z_2, Z_3:

$$Z_1 = 3x_1 + 2x_2 + 5x_3,$$

$$Z_2 = 0.85x_1 + 0.8x_2 + 0.95x_3,$$

$$Z_3 = 0.75x_1 + 0.9x_2 + 0.85x_3$$

s. t.:

$$x_1 + x_2 + x_3 = 1000,$$

$$x_1 \leqslant 500,$$

$$x_2 \leqslant 600,$$

Table 1
Suppliers quantitative information

	Cost	Quality (%)	Service (%)	Capacity
Supplier 1	3	85	75	500
Supplier 2	2	80	90	600
Supplier 3	5	95	85	550

Table 2
The data set for membership functions

	$\mu = 0$	$\mu = 1$	$\mu = 0$
Z_1(net cost)	—	2400	4100
Z_2(quality level)	820	905	—
Z_3(service level)	805	880	—
Demand	950	1000	1100

$$x_3 \leqslant 550,$$

$$x_i \geqslant 0, \quad i = 1, 2, 3.$$

Three objective functions Z_1, Z_2 and Z_3 are cost, quality and service, respectively and x_i is the number of units purchased from the ith-supplier.

The linear membership function is used for fuzzifying the objective functions and demand constraint for the above problem according to steps 1–4. The data set for the values of the lower bounds and upper bounds of the objective functions and a fuzzy number for the demand are given in Table 2.

In Appendix, the membership functions for three objective functions and the demand constraint are provided by which to minimize the total monetary cost and maximize the total quality and service level of the purchased items (step 5).

The fuzzy multiobjective formulation for the example problem is as

Find $x^{\mathrm{T}} = (x_1, x_2, x_3)$

to satisfy:

$$\tilde{Z}_1 = 3x_1 + 2x_2 + 5x_3 \tilde{\leqslant} Z_1^0,$$

$$\tilde{Z}_2 = 0.85x_1 + 0.8x_2 + 0.95x_3 \tilde{\geqslant} Z_2^0,$$

$$\tilde{Z}_3 = 0.75x_1 + 0.9x_2 + 0.85x_3 \tilde{\geqslant} Z_3^0$$

s. t.:

$$x_1 + x_2 + x_3 \cong 1000,$$

$$x_1 \leqslant 500,$$

$$x_2 \leqslant 600,$$

$$x_3 \leqslant 550,$$

$$x_i > 0, \quad i = 1, 2, 3.$$

w_j (j = 1, 2, 3) and β_1 are the weights associated with the jth objective and demand constraint. In this example the assumed DM's relative importance or weights of the fuzzy goals are given as

$w_1 = 0.2$, $w_2 = 0.35$, $w_3 = 0.25$, and the weight of the fuzzy constraint is $\beta_1 = 0.2$.

Based on the convex fuzzy decision-making (32)–(38) and the weights which are given by DM, the crisp single objective formulation for the numerical example is as follows (step 6):

$$\max\, 0.2\lambda_1 + 0.35\lambda_2 + 0.25\lambda_3 + 0.2\lambda_1$$

s. t.:

$$\lambda_1 \leqslant \frac{4100 - (3x_1 + 2x_2 + 5x_3)}{1700},$$

$$\lambda_2 \leqslant \frac{(0.85x_1 + 0.8x_2 + 0.95x_3) - 820}{85},$$

$$\lambda_3 \leqslant \frac{(0.75x_1 + 0.9x_2 + 0.85x_3) - 805}{75},$$

$$\gamma_1 \leqslant \frac{1100 - (x_1 + x_2 + x_3)}{100},$$

$$\gamma_1 \leqslant \frac{(x_1 + x_2 + x_3) - 950}{50},$$

$x_1 \leqslant 500,$

$x_2 \leqslant 600,$

$x_3 \leqslant 550,$

$x_1, x_2, x_3 \geqslant 0.$

The linear programming software LINDO/LINGO is used to solve this problem. The optimal solution for the above formulation is obtained as follows:

$$x_1 = 86,\ x_2 = 386,\ x_3 = 550,$$

$$Z_1 = 3780,\ Z_2 = 904,\ Z_3 = 879.$$

Corresponding to DM's preferences (0.2, 0.35, 0.25, 0.2), in this solution, 550 (maximum capacity) items are assigned to be purchased from supplier 3, because of the highest quality level of supplier 3 performances on the quality criterion. The remaining items are split between supplier 2 and supplier1. The membership function values are obtained as follows:

$$\mu_{z_1}(x) = \lambda_1 = 0.187,\ \mu_{z_2}(x) = \lambda_2 = 0.993,\ \mu_{z_3}(x) = \lambda_3 = 0.992 \quad \text{and} \quad \gamma_1 = 0.72.$$

These values represent that the achievement level of Z_2 (quality) is more than Z_3 and the achievement level of Z_3 (service) is more than Z_1 ($\lambda_2 > \lambda_3 > \lambda_1$). It means that the achievement level of the objective functions is consistent with the DM's preferences ($w_2 > w_3 > w_1$). In other words due to this model, the achievement level of the objective functions correspond with the priority of the purchasing criteria (based on DM's preferences) resulting from the allocated order to each supplier.

In this solution, the degree of achievement of cost objective (λ_1) is obtained as 0.187. In other words, the value of Z_1 is 3780, which is comparable with 2400 ($\mu_{z_1}(x) = 1$). This achievement level may not be enough to satisfy DM in term of the net price objective function. It is realistic in most cases that a poor performance on one criterion can not easily be balanced with a good performance on other criteria. In this case, we can reformulate the presented model, such that the achievement level of membership functions should not be less than an allowed value. The α-cut approach can be utilized to ensure that the degree of achievement for any goals and fuzzy constraints should not be less than a minimum allowed value α. In this case, the weighted additive model should be reformulated by adding new constraints of $\lambda_j \geqslant \alpha$ and $\gamma_r \geqslant \alpha$, $\alpha \in [\alpha^-, \alpha^+]$ to other system constraints. This approach requires that the DM have to choose reasonable values for α to avoid getting infeasible solutions (Chen, 1985; Choobineh, 1993).

In this example, α^- is assumed to be 0.187 and α^+ can be obtained from a Zimmermann's approach (max–min operator) in which all objective functions and constraints are equally important. This value can be calculated by solving the crisp formulation of Zimmermann's approach according to Eqs. (25)–(29); α^+ is equal to the optimal value of the derived solution for λ. In this example, α^+ is calculated at 0.645 and then α can vary from 0.187 to a maximum level of 0.645. Changing α from α^- to α^+, causes the problem solutions to vary from asymmetric to fully symmetric decision making. In this case, α is changing in steps of 0.05, from 0.187 to 0.645. Table 3 presents all optimal solutions S1 to S11 related to these α-cut levels. Fig. 2 represents achievement level variations of membership functions according to α -cut level approach, and Fig. 3 shows allocated quotes to each supplier related to these α-cut levels. Fig. 4 represents the cost criteria and Fig. 5 represents service and quality criteria, in accordance to α -cut level increase in solutions S1 to S11.

The resulting outcomes of utilizing α-cut levels are explained as follows:

Corresponding to DM's preferences, in solution S1, 550 (maximum capacity) items are assigned to be purchased from supplier 3, because of the highest quality level of supplier 3's performance on the quality criterion.

In S1–S6, simultaneously according to DM's preferences and increasing α-cut level from 0.187 to 0.437 it is shown that the purchased values from supplier 3 decreases, and approximately the same values are added to purchasing items of supplier 1, and the quota allocated to supplier 2 is approximately the same value.

Table 3
Optimal solutions S1 to S11 related to α-cut level

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
α-cut level	0.187	0.237	0.287	0.337	0.387	0.437	0.487	0.537	0.587	0.637	0.645
x_1	86	139	192	245	298	351	350	325	301	276	187
x_2	386	390	392	395	398	401	428	464	500	536	600
x_3	550	500	449	400	350	300	273	256	239	222	247
Z_1	3780	3697	3605	3525	3440	3355	3271	3183	3098	3010	2969
Z_2	904.4	905	903	904.3	904.2	904.2	899	890	882.9	874	865
Z_3	879.4	880	878	879.3	879.2	879.2	879	878	878.9	878	880
$\lambda_1 = \mu z_1(x)$	0.187	0.237	0.287	0.337	0.387	0.437	0.487	0.537	0.587	0.637	0.645
$\lambda_2 = \mu z_2(x)$	0.993	1	0.98	0.99	0.99	0.99	0.93	0.83	0.74	0.63	0.645
$\lambda_3 = \mu z_3(x)$	0.992	1	0.97	0.99	0.99	0.98	0.99	0.98	0.98	0.97	1
$\gamma_1 = \mu z_4(x)$	0.77	0.71	0.65	0.6	0.53	0.437	0.487	0.537	0.587	0.637	0.645

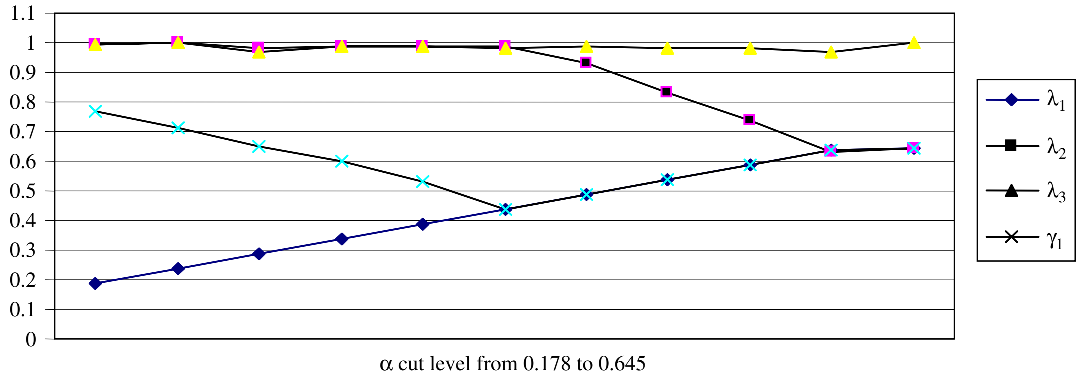

Fig. 2. Degree of achievement objective functions and constraint.

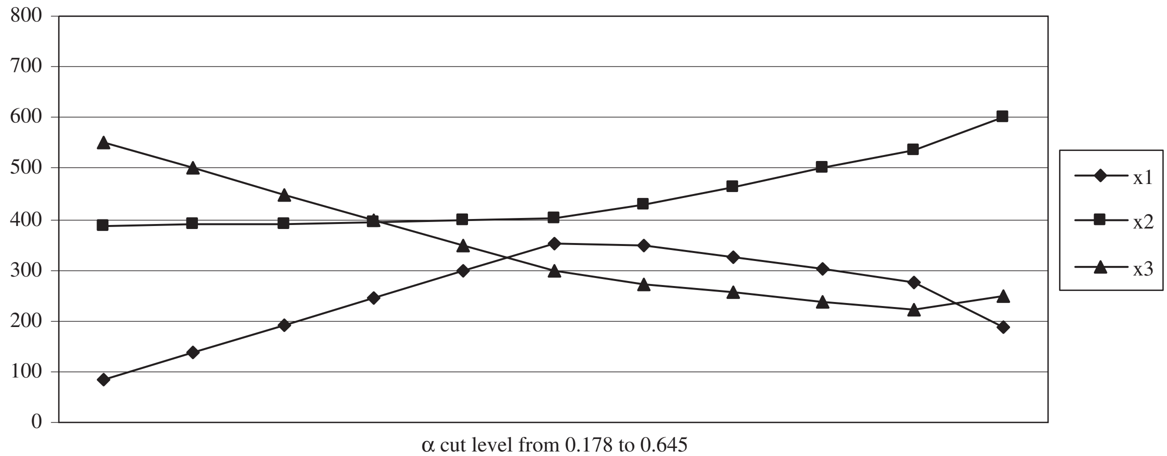

Fig. 3. Quote allocation to each supplier.

S6 represents that the achievement level of all membership functions ($\lambda_2 = 0.99$, $\lambda_3 = 0.98$, $\lambda_1 = \gamma_1 = 0.487$) are more consistent than other solutions to DM's preferences ($w_2 = 0.35$, $w_3 = 0.25$, $w_1 = \beta_1 = 0.2$). In other words, ($\lambda_2 > \lambda_3 > \lambda_1 = \gamma_1$) agrees with ($w_2 > w_3 > w_1 = \beta_1$).

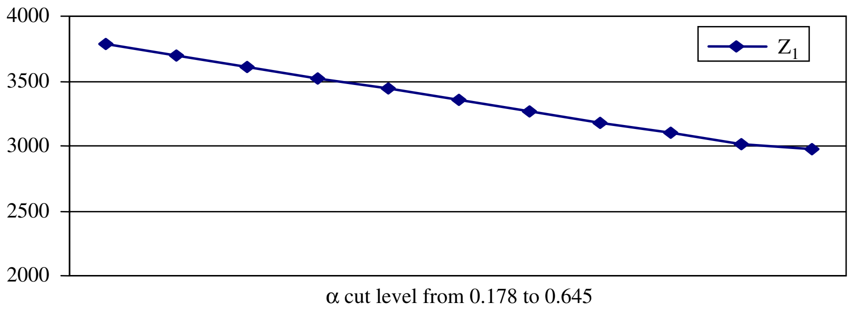

Fig. 4. Objetive function Z_1(net cost).

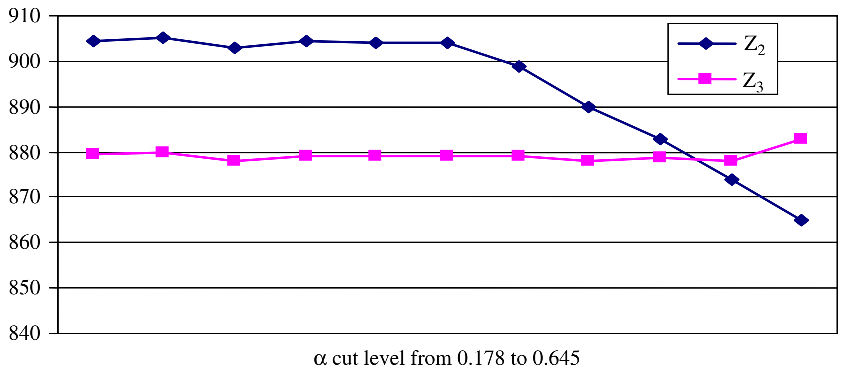

Fig. 5. Objective functions values Z_2 and Z_3.

Table 4
Solutions to numerical example by different approaches

	S6—Reformulated weighted additive	S1—Weighted additive	S11—Zimmermann (weightless)
x_1	351	86	187
x_2	401	386	600
x_3	300	550	247
Z_1(cost)	3355	3780	2969
Z_2(quality)	904	904	865
Z_3(service)	879	879	880

When α-cut level increases from 0.437 to 0.645 (S6–S11), the degree of achievement of all objective functions will be increased and in these situations the quotas allocated to supplier 2 will be increased also, because of the better performance of supplier 2 on cost and service criteria.

According to the DM's preference, quality is the most important criterion. Through Zimmermann's approach, there is no possibility to emphasize on objectives with heavy weights; however, the weighted additive model takes into account the objective's weights. Due to the weighted additive model, the quality performance is improved from 865 to 904 in comparison with the weightless solution. In addition based on the DM's preference, the proposed model has a competence to improve achievement level of membership function objectives or performance on the objectives. Solution S6 from Table 3 indicates that performance on cost criterion improves since it reduces from 3780 to 3355, while quality and service criteria remain at the same level. The results can be summarized in Table 4.

5. Summary and conclusions

Supplier selection is one of the most important activities of purchasing departments. This importance is increased even more by new strategies in a supply chain, because of the key role suppliers perform in terms of quality, costs and services, which affect the outcome in the buyer's company. Supplier selection is a multiple criteria decision-making problem in which the objectives are not equally important. In real cases, many input data are not known precisely for decision-making. For the first time a fuzzy multiobjective model is developed for supplier selection in order to assign different weights to various criteria. This formulation can effectively handle the vagueness and imprecision of input data and the varying importance of criteria in supplier selection problem. The proposed model can help the DM to find out the appropriate ordering from each supplier, and allows purchasing manager(s) to manage supply chain performance on cost, quality, on time delivery, etc. Moreover, through the complete procedure, the fuzzy multiobjective supplier selection problem transforms into a convex (weighted additive) fuzzy programming model and its equivalent crisp single objective LP programming. This transformation reduces the dimension of the system, giving less computational complexity, and makes the application of fuzzy methodology more understandable.

Also in this model, the α-cut approach can be utilized to ensure that the achievement level of objective functions should not be less than a minimum level α. Non-linearity in the supplier selection problem, membership function and fuzzy weights are still open for further investigations.

In a real situation, the proposed model can be implemented as a vector optimization problem; the basic concept is to use a single utility function to express the preference of DM, in which the values of criteria and constraints are expressed in vague terms and are not equally important.

Appendix

The membership functions for three objective functions and the demand constraint (Fig. 6).

$$\text{(a) } \mu_{z_1}(x) = \begin{cases} 1 & Z_1 \leqslant 2400, \\ \frac{4100 - Z_1}{1700} & 2400 < Z_1 < 4100, \quad Z_1 = 3x_1 + 2x_2 + 5x_3, \\ 0 & Z_1 \geqslant 4100. \end{cases}$$

$$\text{(b) } \mu_{z_2}(x) = \begin{cases} 1 & Z_2 \geqslant 905, \\ \frac{Z_2 - 820}{85} & 820 < Z_2 < 905, \quad Z_2 = 0.85x_1 + 0.8x_2 + 0.95x_3, \\ 0 & Z_2 \leqslant 820. \end{cases}$$

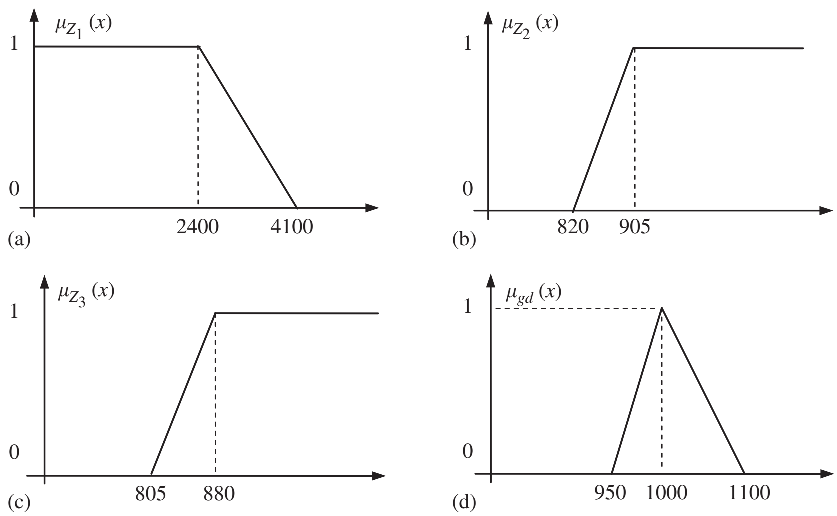

Fig. 6. Membership functions: (a) net costs (Z_1) objective function, (b) quality (Z_2) objective function, (c) service (Z_3) objective function, (d) demand constraint (g_d).

(c) $\mu_{z_3}(x) = \begin{cases} 1 & Z_3 \geqslant 880, \\ \frac{Z_3-820}{85} & 805 < Z_2 < 880, \quad Z_3 = 0.75x_1 + 0.9x_2 + 0.85x_3, \\ 0 & Z_3 \leqslant 805. \end{cases}$

$$d(x) = x_1 + x_2 + x_3$$

(d) $\mu_{g\,d}(x) = \begin{cases} \frac{d(x)-950}{50}, & 950 < d(x) < 1000, \\ \frac{1100-d(x)}{1000}, & 1000 < d(x) < 1100, \\ 0, & d(x) \leqslant 950 \text{ and } (x) \geqslant 1100. \end{cases}$

References

Bache, J., Carr, R., Parnaby, J., Tobias, A.M., 1987. Supplier development systems. International Journal of Technology Management 2 (2), 219–228.

Bellman, R.G., Zadeh, L.A., 1970. Decision making a fuzzy environment. Management Sciences 17, B141–B164.

Chen, S.H., 1985. Ranking fuzzy numbers with maximizing set and minimizing set. Fuzzy Sets and Systems 17, 113–129.

Degraeve, Z., Roodhooft, F., 2000. A mathematical programming approach for procurement using activity based costing. Journal of Business Finance and Accounting 27 (1–2), 69–98.

Dickson, G.W., 1966. An analysis of vendor selection systems and decisions. Journal of Purchasing 2 (1), 5–17.

Dulmin, R., Mininno, V., 2003. Supplier selection using a multi-criteria decision aid method. Journal of Purchasing and Supply Management 9, 177–187.

Erol, I., Ferrel Jr, W.G., 2003. A methodology for selection problems with multiple, conflicting objectives and both qualitative and quantitative criteria. International Journal of Production Economics 86, 187–199.

Gaballa, A.A., 1974. Minimum cost allocation of tenders. Operational Research Quarterly 25 (3), 389–398.

Ghodsypour, S.H., O'Brien, C., 1996. A decision support system for supplier selection using an integrated analytical hierarchy process and operations research methods. Ph.D. Thesis University of Nottingham, Nottingham, UK.

Ghodsypour, S.H., O'Brien, C., 1997. An integrated method using the analytical hierarchy process with goal programming for multiple sourcing with discounted prices. Proceedings of the International Conference on Production Research (ICPR), Osaka, Japan.

Ghodsypour, S.H., O'Brien, C., 1998. A decision support system for supplier selection using an integrated analytical hierarchy process and linear programming. International Journal of Production Economics 56–57, 199–212.

Ghodsypour, S.H., O'Brien, C., 2001. The total cost of logistic in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint. International Journal of Production Economics 73, 15–27.

Holt, G.D., 1998. Which contractor selection methodology? International Journal of Project Management 16 (3), 153–164.

Karpak, B., Kumcu, E., Kasuganti, R., 1999. An application of visual interactive goal programming: a case in vendor selection decisions. Journal of Multi-Criteria Decision Analysis 8, 93–105.

Lai, Y.J., Hawang, C.L., 1994. Fuzzy Multiple Objective Decision Making, Methods and Applications. Springer, Berlin.

Li, C.C., Fun, Y.P., Hung, J.S., 1997. A new measure for supplier performance evaluation. IIE Transactions on Operations Engineering 29, 753–758.

Morlacchi, P., 1997. Small and medium enterprises in supply chain: a supplier evaluation model and some empirical results. Proceedings IFPMM Summer School, August, Saltzburg.

Narasimhan, R., 1983. An analytic approach to supplier selection. Journal of Purchasing and Supply Management 1, 27–32.

Nydick, R.L., Hill, R.P., 1992. Using the Analytic Hierarchy Process to structure the supplier selection procedure. International Journal of Purchasing and Materials Management 28 (2), 31–36.

Roa, C.P., Kiser, G.E., 1980. Educational Buyers' Perceptions Of Vendor Attributes. Journal of Purchasing and Materials Management 16, 25–30.

Sakawa, M., 1993. Fuzzy Sets and Interactive Multiobjective Optimization. Plenum Press, New York.

Soukup, W.R., 1987. Supplier selection strategies. Journal of Purchasing and Materials Management 23 (3), 7–12.

Tiwari, R.N., Dharmahr, S., Rao, J.R., 1987. Fuzzy goal programming-an additive model. Fuzzy Sets and Systems 24, 27–34.

Weber, C.A., Current, J.R., 1993. A multiobjective approach to vendor selection. European Journal of Operational Research 68, 173–184.

Weber, C.A., Current, J.R., Benton, W.C., 1991. Vendor selection criteria and methods. European Journal of Operational Research 50, 2–18.

Zimmermann, H.J., 1978. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems 1, 45–55.

Zimmermann, H.J., 1987. Fuzzy Sets, Decision Making and Expert Systems. Kluwer Academic Publishers, Boston.

Zimmermann, H.J., 1993. Fuzzy Set Theory and its Applications, fourth ed. Kluwer Academic Publishers, Boston.